



Roots of Reason

The Meaning of Fraction Division

$3 \div 1/2$ asks: **how many halves fit in 3?**



1 whole 1 whole 1 whole

$$3 \div \frac{1}{2} = 6$$

six halves fit. count them.

1

Same question as whole numbers

$12 \div 3$ asks how many 3s fit in 12. Nothing changed. Only the size of the piece did.

2

Smaller piece, bigger answer

More small pieces fit than big ones. Dividing by $1/2$ gives an answer **larger** than what you started with.

Division never stopped meaning **how many fit**. Hold onto that question.



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Area Models for Fraction Division

How many $\frac{1}{8}$ s fit in $\frac{3}{4}$?



$\frac{3}{4}$ shaded = 6 eighths

$\frac{1}{4}$ empty

$$\frac{3}{4} \div \frac{1}{8} = 6$$

cut the whole into eighths. count the eighths inside the shading.

1 Recut into the divisor's pieces

Dividing by $\frac{1}{8}$? Cut the model into eighths.
The common cut makes both fractions speak the same language.

2 Then count

$\frac{3}{4} = 6$ eighths. So six pieces of size $\frac{1}{8}$ fit.
The answer is a **count**, not a fraction of anything.

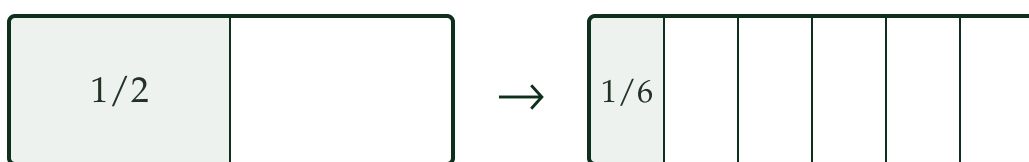
Recut, then count. The model does the dividing for you.



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Sharing a Fraction

$\frac{1}{2}$ a pan of brownies shared by 3 people. How much each?



$$\frac{1}{2} \div 3 = \frac{1}{6}$$

splitting the half into 3 equal shares makes sixths.

1 This division shares

$\div 3$ means split into 3 equal shares. Each new piece is a third of the half, and a third of a half is a sixth of the whole pan.

2 Name pieces from the whole

The answer's name always comes from the **whole pan**, not from the half you started with.

Two questions live inside $\div$: **how many fit**, and **how big is each share**.

Read the problem to know which one it is asking.



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Why the Reciprocal Strategy Works

Flip the divisor and multiply. Here is why that works.

start with one whole

$$1 \div \frac{2}{3} = \frac{3}{2}$$

How many $\frac{2}{3}$ s fit in 1 whole? One full $\frac{2}{3}$ fits, plus half of another. That is $1\frac{1}{2}$, or $\frac{3}{2}$: **the flip of $\frac{2}{3}$.**

then scale up

$$4 \div \frac{2}{3} = 4 \times \frac{3}{2} = 6$$

If $\frac{3}{2}$ of them fit in each whole, then 4 wholes hold 4 times as many. Dividing by a fraction **is** multiplying by its reciprocal.



Check with a model

If the strategy and your drawing disagree, trust the drawing and find the slip.



Estimate first

Dividing by a fraction under 1 should make the answer **grow**. If it shrank, something flipped that should not have.

Use the reciprocal strategy once you can explain it. Not before.